Entrepreneurship and R&D in a Generalized Endogenous Growth Model

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Abstract

We formulate a model that explicitly separates two functions in the growth process: The introduction of new goods and the quality improvement of existing goods. While the latter is performed by the corporate R&D sector, the first is performed by entrepreneurs. We show that in a three sector economy, which also includes a producing sector, there exists a stable non trivial allocation of labor to production, innovation and entrepreneurship. We compute the steady state growth rate of utility in the economy with and without entrepreneurship. The contribution of entrepreneurship on economic growth is shown to be positive.

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It has long been recognized that the
entrepreneurial function is a vital component
in the process of economic growth.
William J. Baumol (1968, p. 65)

1 Introduction

1.1 The Economic Function of Entrepreneurship

While William Baumol emphasized the importance of entrepreneurship nearly 40 years ago, Lazear (2002) claimed more recently that "*The entrepreneur is the single most important player in the economy*." However the role of the entrepreneur has been acknowledged since long.

Walras (1874) (and later Kirzner, 1973), considered the function of the entrepreneur as seeking arbitrage opportunities. As such, the entrepreneur is the driving force behind the tâtonnement process that leads to the general equilibrium in the Walrasian model. Once the equilibrium is attained, however, the entrepreneur is no longer interesting and this is presumably why the usual analysis of equilibria seems independent of the entrepreneurial function.

Keynes (1920, Chapter VI) in his analysis of the recovery of European economy after the Treaty of Versailles considered entrepreneurs as "*the active and constructive element in the whole capitalist society*."

Marshall (1920) considered the entrepreneurial function as equally important as the role of what we know as the production factors. In the 4th book of his *Principles*, he considered four "agents of production": land, labor, capital and organization. He understood "organization" in a structural sense (i.e. in the sense that the notion "industrial organization" reflects) but also in the sense of an activity. Referring to entrepreneurs as "business men" or "undertakers" he states hat:

> *They* [i.e. the entrepreneurs] *"adventure" or "undertake" its risks* [i.e. the risks of production]*; they bring together the capital and the labour required for the work; they arrange or "engineer" its general plan, and superintend its minor details. Looking at business men from one point of view we may regard them as a highly skilled industrial grade, from another as middlemen intervening between the manual worker and the consumer.* Marshall (1920, p. 244)

Hence for Marshall, the function of the entrepreneur is to organize and control the production process and to bear the risks involved with it. This function of the entrepreneur was already implicit in the work of Hawley (1893).[1] Knight (1921) developed on this work and distinguished between (calculable) risk and (incalculable) uncertainty and saw the main function of the entrepreneur in dealing with

[1]Interestingly, even Smith (1776, paragraph I.6.5) acknowledged that entrepreneurs are the agents that affront the risk involved in developing new opportunities, stating that an "*undertaker*" is the one "*who hazards his stock in this adventure*."

the uncertainty that the introduction of new goods to a market, taking the responsibility and the control for this activity, implies. Hence Knight expanded the Marshallian function of entrepreneur by explicitly linking it to the introduction of new goods. And once more, once production is organized and running smoothly, the entrepreneurial function fades and profit maximization takes over.

Schumpeter (1911, 1942) pushed the idea of a central role for the entrepreneur in capitalist economies. He saw the function of the entrepreneur in the "*recognition and realization of new economic opportunities*", where opportunities were not only potential products but also potential production processes and opportunities in marketing and reorganization. By considering novelty as driver of opportunity, the notions risk and uncertainty are of course implicitly part of the entrepreneurial function. Hence, in summary, this literature considers entrepreneurs as agents who seek opportunities in form of arbitrage or potential innovations, who organize and control the exploration of this opportunity and who are willing to bear the risk of doing so. In short they are the agents of change.

In an innovation oriented or knowledge based economy, the function of opportunity recognition and taking the risk of realizing it becomes more prominent. The act of the entrepreneur is no longer a short preface to static equilibrium but a driving force in shaping the dynamics in the economic system. Baumol (2002b) distinguishes this entrepreneurial function explicitly from the role of larger incumbent corporations who are rather involved into routine processes of large scale innovation. These processes seem quantitatively more important as they are easier to measure. R&D expenditure and the number of patents generated are larger and so are the resulting job creation and value added. However, a number of systematic studies have provided evidence that breakthroughs and new products are rather introduced by small and young firms, i.e. by entrepreneurs.[2] In that sense Baumol (2002b) refers to innovation as an integrated process based on a division of labor between small firms, who launch new products and introduce new technologies, and large firms, who take on these ideas and develop them. Hence entrepreneurial firms and large firms coexist in what Baumol (2002a) calls a "*David-Goliath Symbiosis*". In that respect, entrepreneurship plays an important role for the economic dynamics and for the growth process of an economy. Failing to understand entrepreneurship is failing to understand growth.

1.2 Entrepreneurship as a Conduit for Knowledge Spillovers

The traditional model of economic development (Solow, 1956) formalizes economic growth as the result of accumulating production capital through investment. The

[2] e.g. Scherer (1980) or CHI Research Inc. (2002). The U.S. Small Business Administration (1995, p.114) enumerates some 70 important innovations by small firms in the 20th century, ranging from low-tech innovations such as the Zipper or Bakelite to high-tech ones such as the Nuclear Magnetic Resonance Scanner or the Microprocessor.

failure to explain US growth using that model, however, made Solow himself conclude that productivity growth through innovation, not capital accumulation, must be the engine of long-run growth in modern economies. The endogenous growth theory (Romer, 1986; Lucas, 1988; Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) explicitly models the creation of innovations by introducing a dedicated knowledge generating sector (R&D or education). One of the main assumptions underlying this theory is that knowledge behaves like a public good, i.e. it is non-exhaustive and non-excludable. This implies that the stock of existing knowledge and the newly created knowledge is available (i.e. spills over) automatically to all economic agents. In that respect, the properties of knowledge differ fundamentally from the "traditional" production factors, i.e. capital and labor.

The public goods assumption implicitly suggests that all new knowledge is fully commercializable and fully applied to the production process. However, as Arrow (1962) pointed out, new knowledge differs from the traditional production factors not only by its public goods characteristics, it is also inherently uncertain. By uncertainty, Arrow understood the fact that it is a priori unknown if newly generated knowledge can be transferred successfully into a viable innovation, be it a new product or any other innovation. Indeed, one can think of the stream of new knowledge arriving at a certain time period as involving different levels of risk. For some of the new knowledge, its usefulness, hence the possibility of transforming it into a new product is obvious to agents involved in the production process. Think e.g. of quality improvements of existing products. On the other end of the "uncertainty scale" is new knowledge whose usefulness is not obvious at all, i.e. this knowledge is rather distant from existing products. Here, we can think of new knowledge that can be either very useful, indeed potentially revolutionizing[3], or useless, indeed potentially inapplicable. This means that with increasing uncertainty of the new knowledge, the *variance* of the value of new knowledge increases.

On the individual level, this implies that an entrepreneur develops a vision on opportunities that are based on the untapped part of the public available knowledge. If the entrepreneur guesses that the potential returns to those products are superior to what he would earn as an employee, he will engage into starting up a new venture to realize his vision. By doing so, he explores new knowledge that otherwise would remain unexplored, he thus is part of the knowledge spillover process in the economy (Audretsch, Keilbach and Lehmann (2006) denote this process the *Knowledge Spillover Theory of Entrepreneurship*).

Summarizing this discussion, we state that the function of the entrepreneur is the

[3] In the extreme case, these are technologies that are able to start up a new innovation life cycle in the sense of Gort and Klepper (1982).

one who seeks arbitrage and innovation opportunities and is prepared to pursue these opportunities and to bear the risk involved in this enterprise. Although this importance has widely been acknowledged, the entrepreneur is absent in formal models of growth, even in models of innovation driven growth. In that respect it is interesting to observe that Baumol (1968) states that

> *In recent years, while the facts have apparently underscored the significance of his* [the entrepreneur's] *role, he has at the same tame virtually disappeared from the theoretical literature.* Baumol (1968, p. 64)

In this paper we present a model in which the entrepreneurial function is made central to the process of economic growth. The entrepreneurs, however, do not drive R&D (or education) off the stage. Instead they can be positioned clearly between knowledge, that for simplicity is assumed to evolve gradually and autonomously, and product improvements, that are the domain of profit driven corporate R&D workers. Entrepreneurs in our model are the agents that combine ideas from the knowledge stock into opportunities and then bring new products to the market. The common knowledge stock also benefits quality improving R&D. Section 2 presents the model. Section 3 analyses the implications of the model and highlights the role of entrepreneurs by comparing equilibrium dynamics with and without entrepreneurial activity. It is shown that sustainable growth does not require entrepreneurs but is greatly enhanced by it. Section 4 concludes.

2 The Model

We consider an economy that is composed of three aggregates, i.e.

$$Pop = L + R + N \tag{1}$$

where Pop is the population involved in the economic process, L is labor involved in production, R is employees involved in R&D and N is individuals acting as entrepreneurs.

2.1 Producing Sector

Laborers produce n diversified and existing products. Each product i has a certain quality q_i assigned and comes with a corresponding price p_i and consumption level c_i. Hence, on the basis of a standard Dixit-Stiglitz love-of-variety instant utility function, augmented with variety specific quality parameter consumers solve

$$\max_{c_i} : \left(\int_0^n q_i^{1-\alpha} c_i^{\alpha} di \right)^{1/\alpha} \quad \text{s.t.} \quad \int_0^n c_i p_i di \leq E, \tag{2}$$

with $0 \leq \alpha \leq 1$. E is expenditure on consumption. It can be verified in the utility function that economic growth can come from 3 distinct sources. Variety expansion, quality improvement and regular increases in consumption volumes increase the utility index over time. To derive the instant global demand functions for all current and future goods in this CES-utility function is straightforward:[4]

$$c_i^D = q_i \left(\frac{p_i}{P}\right)^{\frac{1}{\alpha-1}} \frac{E}{P} \quad \text{where} \quad P \equiv \left(\int_0^n p_i^{\frac{\alpha}{\alpha-1}} q_i di\right)^{\frac{\alpha-1}{\alpha}}, \tag{3}$$

where P is a quality adjusted exponentially weighted price index that can be defined a the minimum cost of one util.

Production takes place under monopolistic competition such that producers can set prices. At every point in time they take demand and the quality of their product as given. Hence producers solve

$$\max : \pi_i = c_i \cdot p_i - w \cdot l_i \tag{4a}$$
$$s.t. : c_i = c_i^D \tag{4b}$$
$$s.t. : y_i = bl_i, \tag{4c}$$

π_i being profits of firm i, w is the wage level and l_i is the labor force employed by i to produce c_i. (4b) makes sure that the market clears and (4c) is a production function with labor as single input. This condition excludes the possibility for steady state growth from increases in production volumes as the productivity parameter is given and the level of employment fixed by the absence of population growth in the model. Solving the set of equations (4) yields the equilibrium price of product i

$$p_i = \frac{w}{\alpha b} \tag{5}$$

and the equilibrium profit of producing it

$$\pi_i = \frac{(1-\alpha)Eq_i}{nQ} \quad \text{where} \quad Q \equiv \frac{1}{n}\int_0^n q_i di. \tag{6}$$

Equation (6) makes clear that the profit of product i, π_i, increases with its quality q_i. Positive profits will create an incentive to enter the market, i.e. to propose a new product with a given initial quality and with unknown demand. We denote agents that do so as entrepreneurs.

2.2 Knowledge, R&D and Entrepreneurship

Consider K, the level of the existing body of knowledge in the economy. As we are not primarily interested in the sources of growth but rather want to focus on the role of the entrepreneur in economic growth, we assume this rate to be exogenously given by g.[5] Knowledge has two impacts in our model.

[4] See for example Grossman and Helpman (1991).

[5] In another version of the model, we endogenized g as a positive function of R in a specification à la Romer (1990). The outcome of the model is not affected by this specification. We therefore keep g exogenous for tractability.

First, knowledge positively affects the increase of quality of existing products in the R&D process. We specify this activity as

$$\dot{q}_i = h(KR_i)^{\gamma} \tag{7}$$

where $0 \geq \gamma \geq 1$. K is the existing body of knowledge in the economy that augments R_i, the level of R&D effort in firm i. The marginal productivity of effective R&D is decreasing in the level of R&D (expressed by R_i) itself. Note also that the rate of quality improvement inevitably decreases in the level of quality achieved. Quality improvement is thereby effectively excluded as a source of steady state growth in this equation. The rate of labor augmentation that emanates from exogenous knowledge growth is exactly offset by the spreading of a given number of R&D workers over a growing number of firms. The result is a constant increment in quality that vanishes in relative terms. Equation (7) thus deviates from standard quality ladder models, where a given level of effort yields a constant rate of quality improvement, but this assumption can be justified as improving quality on already high quality products is typically harder than thinking up quality improvements on low quality products.

The second role of knowledge in our model is to determine the number of potential products n^P. Consider n^P as the number of *opportunities* that can be developed out of the current state of knowledge K. Hence

$$n^P = \xi K \tag{8}$$

Opportunities include unrealized as well as realized products, i.e. $n \subset n^P$. However, as long as $n < n^p$, there exist unexploited opportunities hence room for entrepreneurial activity. By the act of starting a new venture, an entrepreneur introduces a new variety of a new product in the market. Hence he is developing a previously unrealized idea out the pool of potential products n^P. Formally this activity can be represented by:

$$\dot{n} = a(n^P - n)N^{\beta} \tag{9}$$

where $0 \geq \beta \geq 1$. N is the level of entrepreneurial activity. Note that this introduces strong diminishing returns with respect to n as the marginal productivity of the entrepreneur falls to 0 when all opportunities are exploited, i.e. if $n^P - n \rightarrow 0$. Equation (9) therefore implies that variety expansion in the model is restricted in the long run to the rate at which knowledge expands (at rate g, hence through equation 8).

The fact that profits are made by incumbent firms implies that there is an incentive to enter the market, i.e. to start up a new venture. This implies that a new variety is introduced to the market i.e. n increases. The value of realizing one commercial opportunity is given by the expected discounted profit flow that the product i yields at some initial quality level q_{i0}, which can be written as:

$$v_n(t) = \int_t^{\infty} e^{-r(\tau - t)} \pi_i(q_{i0}, \tau) d\tau. \tag{10}$$

q_{i0} will be realized only when the product is first introduced and is therefore ex ante unknown to the entrepreneur. From equation (6) we see that increasing variety, n, and average quality, Q, erode the profits of firms. Moreover, it is unknown ex ante how fast competitors will improve the relative quality of their products, how fast variety expands and how fast consumption expenditure grows. Hence the rate of profit erosion is unknown.

For now we abstract from any uncertainty that is inherent to the introduction of new products to the market to illustrate the essential mechanisms in the model. Hence we assume that q_{i0} is a known parameter and entrepreneurs form rational expectations, which implies that they have on average correct expectations on future profit erosion rates. As we will show below, they are constant in steady state equilibrium and hence rational expectations imply that entrepreneurs expect constant profit erosion rates. In that case we show in appendix A on page 14 that the marginal value of a business opportunity (10) can be rewritten to:

$$v_n(q_{i0}, t) = \frac{\pi_i(q_{i0}, t)}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n} = \frac{(1-\alpha)EQ^{-1}q_{i0}n^{-1}}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n}. \tag{11}$$

Equation (11) states that the instant profit flow π_i is discounted against the interest rate plus the rate of average quality improvement plus the introduction rate of new products by entrepreneurs, which by (6) is equal to the rate of profit erosion due to variety expansion and quality improvements in substitutes. This value is not augmented by the fact that an incumbent can improve his own quality parameter because we assume this is not costless. In other words, in equilibrium the investments required to make such quality improvements will exhaust (exactly) the additional discounted profits that result from such improvements.

This assumption and the assumption on limited variety expansion serve to tie down the steady state growth rate. No sustainable long run growth is possible without variety expansion which is impossible without an expanding set of opportunities and knowledge base and the entrepreneurial activity that converts opportunities into realities. It thus implies that entrepreneurship is an essential contributor to long term growth in the model, even if it cannot be labeled the ultimate source (which is exogenous knowledge expansion). As we do not aim to explain growth itself but rather illuminate the role of entrepreneurs in transforming knowledge accumulation into economic development, we feel that these assumptions are justified in the context of our model. See Sanders (2005) on extensions that endogenize the growth of the knowledge stock itself.

The value of adding to the quality index at the margin is given by the derivative of (11) with respect to q_i. As the effect of one product's quality index on Q is negligible we obtain:

$$v_q(q_i, t) = \frac{1}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n} \frac{d\pi_i(q_i, t)}{dq_i} = \frac{(1-\alpha)EQ^{-1}n^{-1}}{r - \dot{E}/E + \dot{Q}/Q + \dot{n}/n}. \tag{12}$$

2.3 Equilibrium

The equilibrium in the model requires the market clearing conditions for labor, entrepreneurship and research. If we assume that the opportunity costs for entrepreneurs and R&D workers are given by the general wage level, we can calibrate the productivity parameters a, b and h to obtain a reasonable allocation of labor over the various activities in the economy. Of course that implies that entrepreneurs, R&D workers and production workers are perfect substitutes, which we certainly to not whish to claim. Still, as long as we assume that the wage in production provides the opportunity costs to R&D workers and entrepreneurs and there is free entry in both occupations, the result is similar as the production wage puts a floor in the marginal revenue of engaging in entrepreneurial activity and doing research. Therefore this assumption is the best we can do in a model with homogeneous labor.

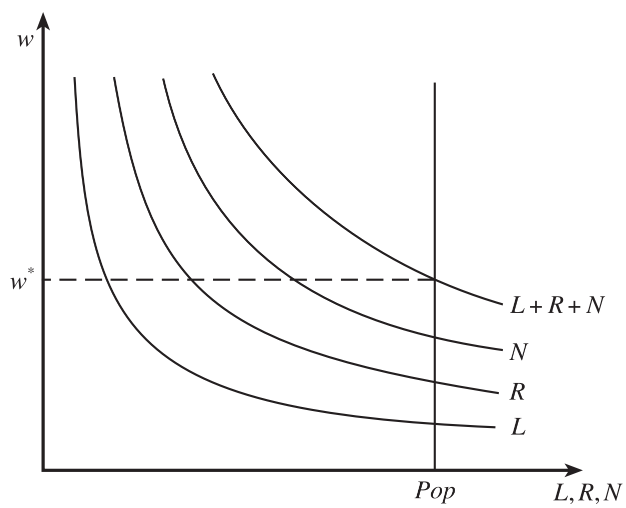

Figure 1: Equilibrium wage level

Having made that point, the labor market clears when equation (1) is fulfilled. The demand for labor can then be derived from inverting production function (4c), substituting for quantities using demand in (3) and prices in (5) yields:

$$l_i = \frac{1}{n} \frac{\alpha E}{w} \frac{q_i}{Q}. \tag{13}$$

Integrating over all n yields the aggregate labor demand for production:

$$L = \frac{\alpha E}{w} \tag{14}$$

To obtain the demand for entrepreneurship and R&D, the marginal value product value of these activities is set equal to the wage level w. Assuming all entrepreneurs expect to enter the market at the same quality level and rearranging yields:

$$N = \left(\frac{w}{a\beta}\right)^{\frac{1}{\beta-1}} v_n(q_{i0}, t)^{\frac{1}{1-\beta}} (\xi K - n)^{\frac{1}{1-\beta}} \tag{15}$$

It is worth noting that entrepreneurial activity is negatively related to the wage in production and positively related the the marginal value of a business opportunity v_n. From (11) we know that the equilibrium level of N increases with the profit level and with the growth rate of E but decreases with the growth rate of Q and n. In addition N responds positively to increases in the knowledge stock. The intuition is that more knowledge makes entrepreneurs more likely to succeed. Similarly, the demand for R&D employees we find for incumbent firm i:

$$R_i = \left(\frac{w}{h\gamma}\right)^{\frac{1}{\gamma-1}} v_q(q_i, t)^{\frac{1}{1-\gamma}} K^{\frac{\gamma}{1-\gamma}}$$

which, integrated over n yields:

$$R = \left(\frac{w}{h\gamma}\right)^{\frac{1}{\gamma-1}} v_q(q_i, t)^{\frac{1}{1-\gamma}} n K^{\frac{\gamma}{1-\gamma}}. \tag{16}$$

Hence the level of R&D activity is also negative in the general wage level and positive in the knowledge stock. Also it responds positively to higher expenditure growth and negatively to increases in the average quality level. The level of R&D activity, however, we know from (12) that R does not does not respond to the level of profits but to the marginal increase in profit that a quality improvement allows. In addition, the number of existing varieties now has a positive impact as more varieties imply more varieties that need quality enhancing R&D.

As (14), (15) and (16) are all decreasing in the wage, there is a unique wage level that clears the labor market. This can be illustrated in a figure such as Figure 1. Due to the different elasticities of the curves, however, it is not possible to compute the analytical solution for the equilibrium wage. To prove the existence of a steady state equilibrium, however, this is not required. We can infer its existence from assuming its properties and proving that these assumption yields a stable equilibrium in the labor market.

2.4 Steady State

For the steady state to be stable, the allocation of Pop to the aggregates L, R and N must be stable. Hence this implies that

$$\frac{\dot{L}}{L} = \frac{\dot{R}}{R} = \frac{\dot{N}}{N} = 0. \tag{17}$$

Given that the population does not grow, the level of production, research and entrepreneurship have to be constant in the steady state. Taking time derivatives and computing the growth rate of (14), (15) and (16) yields the following conditions for the steady state:

$$\frac{\dot{E}}{E} = \frac{\dot{w}}{w} \tag{18a}$$

$$\frac{\xi\dot{K} - \dot{n}}{\xi K - n} = \frac{\dot{Q}}{Q} + \frac{\dot{n}}{n} \tag{18b}$$

$$\gamma\left(\frac{\dot{K}}{K} - \frac{\dot{n}}{n}\right) = \frac{\dot{Q}}{Q} \tag{18c}$$

Using the fact that in any steady state the growth rate of n (the number of varieties) must be equal to the growth rate of n^P (the number of opportunities) and therefore equal to the growth rate of K (the knowledge stock), implying that the difference between K and n grows at the same rate g. Combining this with the set of equations (18) yields the results that a steady state may exist as long as

$$\frac{\dot{w}}{w} = \frac{\dot{E}}{E} = r - \rho; \quad \frac{\dot{K}}{K} = \frac{\dot{n}^P}{n^P} = \frac{\dot{n}}{n} = g \quad \text{and} \quad \frac{\dot{Q}}{Q} = 0. \tag{19}$$

Normalizing the expenditure for one unit of utility to $E = 1$ yields the growth rate of utility in the economy

$$g_U = -\frac{\dot{P}}{P} = \frac{1-\alpha}{\alpha} g. \tag{20}$$

Given conditions (19) for the steady state, we derive from equation (9) that the total level of entrepreneurial activity in the steady state, N^{SS}, must be equal to

$$N^{SS} = \left(\frac{g}{a}\frac{n}{n^P - n}\right)^{1/\beta}, \tag{21}$$

which states that N^{SS} increases in g, the growth rate of knowledge, and therefore the growth rate of potential products (from equations 19). On the other hand, the level of the steady state of N increases as n approaches n^P. The interpretation is that as the number of new varieties in the model approaches its maximum value, the level of entrepreneurship has to increase to maintain the equilibrium rate of $\dot{n}$ (from 9).

In a similar way, we derive from equation (7) the steady state level of R as

$$R^{SS} = \frac{n}{K}\left(\frac{gQ}{h}\right)^{1/\gamma} \tag{22}$$

The steady state demand for labor, L^{SS} can then be derived from (1). Figure 2 gives a graphic representation of the steady state distribution.

2.5 Dynamic Properties of the Steady State

To analyze the dynamic properties of the model, we derive the equations of motion for Q, n and K. The formal derivation of these equations is given in Appendix C. There we show, that the system will converge to a stable, non trivial equilibrium in

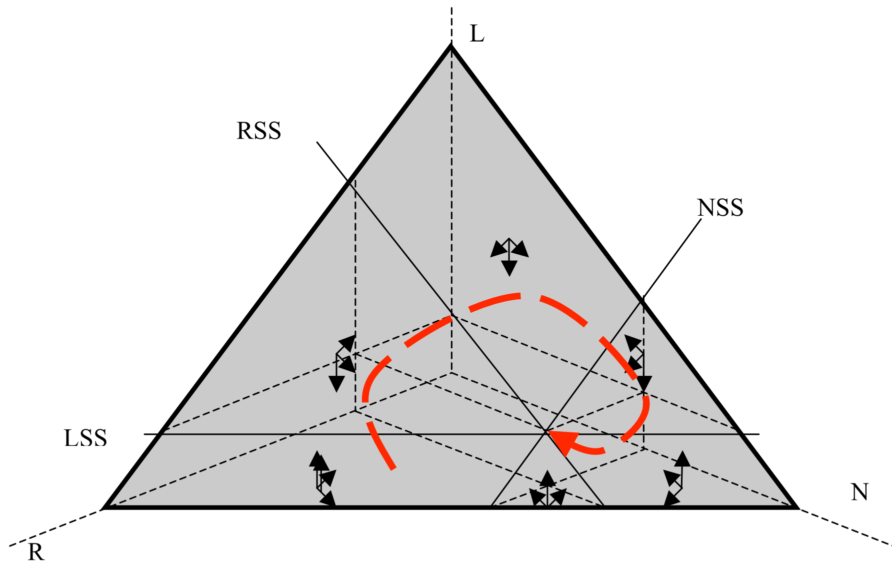

Figure 2: Dynamics and Steady State Distribution in the Labor Market on the Unit Simplex

the $(Q, n/K)$ space, i.e. a point with positive long term growth rate of utility and with a positive average quality of the products in the economy. Figure 3 depicts this dynamic equilibrium.

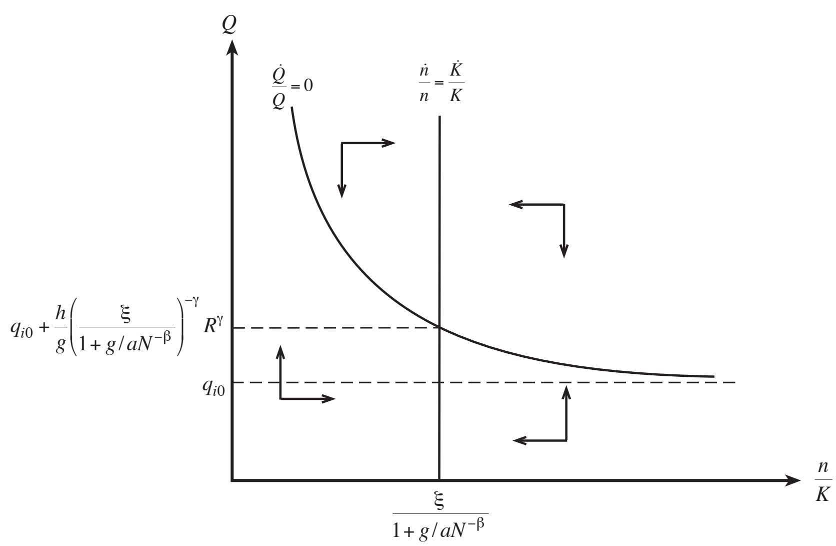

Figure 3: Phase Space of the Dynamics in the Model

3 Assessing the Contribution of Entrepreneurship to Growth

To asses the contribution of Entrepreneurship to growth in our model, we set $q_{i0} = 0$ such that in equilibrium $N = 0$ and therefore (from equation 9) $\dot{n} = 0$, hence no new product variety will be introduced in the economy. However (17) still defines the steady state of the model. Also the demand for R&D is still given by (16). Given that now $\dot{n}/n = 0$, taking time derivative of (16) and requiring them to equal 0 yields the following conditions for the steady state

$$\frac{\dot{E}}{E} = \frac{\dot{w}}{w} \tag{23a}$$

$$\gamma\left(\frac{\dot{K}}{K}\right) = \frac{\dot{Q}}{Q} \tag{23b}$$

Solving for the steady state of the level of R&D, we obtain

$$R^{SS} = \frac{n}{K}\left(\frac{\gamma g Q}{h}\right)^{1/\gamma} \tag{24}$$

which is a constant since K is assumed to grow at g and therefore (by 23b), Q grows at $\gamma \cdot g$.[6] Again normalizing $E = 1$ we obtain

$$g'_U = \frac{1-\alpha}{\alpha}\gamma g, \tag{25}$$

which is smaller than (20) by a factor γ, the output elasticity of the R&D industry. Hence the contribution of Entrepreneurship to economic growth is

$$(1-\gamma)\frac{1-\alpha}{\alpha}g,$$

which is always positive for $g > 0$ and $\gamma < 1$. Note that with $N = 0$, the mechanism of growth in the model has shifted from variety expanding growth to purely quality enhancing growth, however both driven by knowledge accumulation.

4 Summary and Conclusions

While it is acknowledged in the literature that entrepreneurship plays an important role in the process of economic growth, existing growth models do not explicitly consider this the entrepreneur's function. In this paper, we model economic growth as a function of two distinct innovation processes, variety expansion, i.e. the introduction of new products, and quality enhancement of existing products.

[6]Note that this implies that the growth of the model *without* entrepreneurship is entirely driven by quality improvement other than in the model *with* entrepreneurship, where growth is driven by variety expansion and where quality improvement converges to 0 (equation 19). Hence the corner solution of the model where $N = 0$ yields a qualitatively different outcome.

While the latter function is ascribed to an R&D sector that consists of existing firms, the first function is executed by entrepreneurs.

From a model building point of view, we show that it is possible to integrate both types of innovation process into a growth model. The model degenerates into a standard endogenous growth model with quality enhancement if entrepreneurs so not exist. On the other side, the model degenerates to a standard model with variety expansion if the R&D sector does not exist. Therefore, our model can be considered as a *generalized endogenous growth model.* To our knowledge, this has not been done previously.

Based on this model, we can show that the economy converges to a stable non-trivial distribution among the three aggregates of the working population: labour, R&D employment and entrepreneurship. We can also show, that this distribution leads to a stable non-trivial path of steady state growth of utility in the economy. We finally show formally that entrepreneurship does make a positive contribution to the process of growth of utility.

Based on these findings, it can be argued that scarcity of entrepreneurial talent and/or adequately trained R&D workers will slow down an economy. If, for some reason, entrepreneurial activity falls short of its steady state level, the level of R&D activity will actually be too high as the rate of variety expansion is below its steady state level and this increases the value of quality improvement. On the other hand, a lack of R&D capacity will cause a lower rate of aggregate quality improvement, making entrepreneurial activity artificially attractive.

As the US and Europe can both access the same pool of knowledge, respective relative shortages of R&D capacity and entrepreneurial spirit may explain the apparent specialization in entrepreneurial and corporate innovation respectively. It can also be verified in the model that increasing ξ, i.e. improving the permeability of the knowledge filter generates a one time increase in growth and raises the economic to a higher level of utility but not to a permanent increase in the growth rate.

A From Equation (10) to Equation (11)

Entrepreneurs, when introducing a new product to the market, face uncertainty. In our model we have assumed symmetry among products in the utility function, which implies there is always a positive, actually infinite, demand for new varieties. Of course this is a simplification and uncertainty runs deeper than even our model allows. Yet a lot of uncertainty can still be introduced if required. We abstract from doing so to work out the fundamental properties of the model, but in no way would want to claim that our formalization of the entrepreneurial act captures this fundamental aspect of it. Having said that let us proceed. The reward for a successful entrepreneur is the flow of rents, monopoly profits, he can earn by bringing his new product variety to the market. Discounted to the present this flow of profits is given by the expression:

$$v_{n+1}(t) = \int_0^\infty e^{-r(\tau-t)} \pi_{n+1}(q_{n+1,0}, \tau) d\tau$$

By the assumed symmetry of goods and by assuming a known and given initial quality level for all new goods, this can be written as in equation (11):

$$v_n(t) = \int_0^\infty e^{-r(\tau-t)} \pi_i(q_{i0}, \tau) d\tau$$

Recall from (6) that profits, without further quality improving investments, are given by:

$$\pi_i(t) = \frac{(1-\alpha)E(t)q_i(t)}{n(t)Q(t)} = \frac{(1-\alpha)E(t)q_{i0}}{n(t)Q(t)}$$

Quality improvements are costly in our model so once in operation the decision to invest in them is a new and separate decision. The (discounted) additional profits of such investments will, in equilibrium, just offset the costs and therefore have no impact on the decision to bring the product to the market. Having said that, it is clear that the growth rate of profits equals:

$$\frac{\dot{\pi}_i(t)}{\pi_i(t)} = \frac{\dot{E}_i(t)}{E_i(t)} - \frac{\dot{n}_i(t)}{n_i(t)} - \frac{\dot{Q}_i(t)}{Q_i(t)}$$

Which entrepreneurs with rational expectations know will be constant in the steady state. But is the growth rate is expected to be constant, profits at time t are given by:

$$\pi_i(q_{i0}, t) = e^{\left(\frac{\dot{E}_i(t)}{E_i(t)} - \frac{\dot{n}_i(t)}{n_i(t)} - \frac{\dot{Q}_i(t)}{Q_i(t)}\right)(t-t_0)} \pi_i(q_{i0}, t_0)$$

Dropping the time arguments on constant growth rates allows us to write the integral as:

$$v_n(t) = \int_0^\infty e^{-r(\tau-t)} e^{\left(\frac{\dot{E}}{E} - \frac{\dot{n}}{n} - \frac{\dot{Q}}{Q}\right)(\tau-t)} \pi_i(q_{i0}, \tau) d\tau$$

Which solves easily into (11) Q.E.D.

Similar reasoning applies to (12), where the value of increasing quality is equal to the discounted marginal profit from higher quality

B Stability of the Steady State Allocation of Labor

The stability of the steady state can be shown by deriving the sign of the impact on entrepreneurial activity N and research and development R of rising n and Q out of steady state equilibrium. The intuition is straightforward. If N exceeds its steady state level, the growth rate in n, by equation (10) is also higher than its steady state value. This implies that the economy will return to steady state only if a rise in n reduces the deviation from equilibrium entrepreneurial activity. Formally we check:

$$\frac{d(N(t)-N^{SS})}{dn(t)} < 0.$$

Similarly for R&D:

$$\frac{d(R(t)-R^{SS})}{dQ(t)} < 0.$$

If these conditions hold we know that production labor is also adjusting in the right direction and the steady state is stable. Substituting for $N(t)$ using equations (15) and (11) and for N^{SS} using (21) we find:

$$N(t)-N^{SS} = \left(\frac{w}{a\beta}\right)^{\frac{1}{\beta-1}} (\xi K - n)^{\frac{1}{1-\beta}} \left(\frac{(1-\alpha)Eq_{i0}}{nQ}\right)^{\frac{1}{1-\beta}} \left(r - \frac{\dot{E}}{E} + \frac{\dot{n}}{n} + \frac{\dot{Q}}{Q}\right)^{\frac{1}{\beta-1}} - \left(\frac{R^{\phi}}{a}\frac{n}{\xi K - n}\right)^{\frac{1}{\beta}}.$$

The derivative with respect to n is given by:

$$\frac{d(N(t)-N^{SS})}{dn} = \frac{\left(\frac{aEq_{i0}(1-\alpha)\beta(\xi K-n)}{n^2 Q\left(r-\frac{\dot{E}}{E}+\frac{\dot{n}}{n}+\frac{\dot{Q}}{Q}\right)w}\right)^{\frac{1}{1-\beta}}(n-2\xi K)\beta + \left(\frac{R^{\phi}n}{a(\xi K-n)}\right)^{\frac{1}{\beta}}(\beta-1)K\xi}{n(1-\beta)\beta(\xi K-n)},$$

of which the denominator is larger than 0, as well as the large terms between brackets in the numerator. It is then easily verified that the derivative is negative and hence the number of entrepreneurs will return to the steady state level when the economy finds itself out of equilibrium.

Similarly we can present the derivative of R&D employment with respect to average quality levels:

$$\frac{d(R(t)-R^{SS})}{dQ} = -\frac{n}{Q}\left(\left(\frac{R^{\phi}Q}{h}\right)^{\frac{1}{\gamma}} K\gamma + \frac{1}{1-\gamma}\left(\frac{EhK^{\gamma}(1-\alpha)\gamma}{n^2 Q\left(r-\frac{\dot{E}}{E}+\frac{\dot{n}}{n}+\frac{\dot{Q}}{Q}\right)w}\right)^{\frac{1}{1-\gamma}}\right)$$

Again it is easily verified that this expression is smaller than 0, which establishes the stability of the steady state as illustrated by the arrows in Figure 1.

C Dynamic Properties of the Model

Equation (6) states the *index of average quality* of all products in the economy

$$Q(t) \equiv \frac{1}{n(t)} \int_0^{n(t)} q_i(t)di.$$

Deriving this index with respect to t yields

$$\begin{aligned}\frac{dQ(t)}{dt} &= -\frac{\dot{n}(t)}{n(t)}Q(t)+\frac{\int_0^{n(t)}\dot{q}_i(t)di+q_{i0}\dot{n}(t)}{n(t)}\\ &= (q_{i0}-Q)\frac{\dot{n}}{n}+\bar{\dot{q}}_i=0 \text{ for } \dot{q}_i=\bar{\dot{q}}_i\forall i=1,2,\ldots,n\end{aligned}$$

Substituting in the R&D quality improvement function (7) we have:

$$h\left(\frac{n}{K}\right)^{-\gamma}R^{\gamma}=(Q-q_{i0})\frac{\dot{n}}{n}$$

Solving for Q and substituting g for the growth rate of n yields:

$$Q=q_{i0}+R^{\gamma}\frac{h}{g}\left(\frac{n}{K}\right)^{-\gamma}.$$

It can be verified that $Q(t)$ will increase over time if $Q(t)$ lies below this line: Let

$$\begin{aligned}\dot{Q}(t) &> 0\\ (q_{i0}-Q(t))\frac{\dot{n}(t)}{n(t)}+h\left(\frac{n(t)}{K(t)}\right)^{-\gamma}R^{\gamma} &> 0\\ h\left(\frac{n(t)}{K(t)}\right)^{-\gamma}R^{\gamma} &> (Q(t)-q_{i0})\frac{\dot{n}(t)}{n(t)}\\ \frac{h}{g}\left(\frac{n(t)}{K(t)}\right)^{-\gamma}R^{\gamma}+q_{i0} &> Q(t)\end{aligned}$$

Now consider the condition for n/K to be stable. Required is:

$$\dot{n}/n=\dot{K}/K=g$$

By the entrepreneurial production function (9) we know that:

$$\frac{\dot{n}(t)}{n(t)}=\frac{a(n^P(t)-n(t))N^{\beta}}{n(t)}=\frac{a(\xi K(t)-n(t))N^{\beta}}{n(t)}=\frac{a\xi K(t)N^{\beta}}{n(t)}-\frac{aN^{\beta}}{n(t)}=g$$

Solving for n/K yields:

$$\frac{n(t)}{K(t)}=\frac{\xi}{1+g/aN^{-\beta}}$$

If n/K exceeds this value, n will grow at a rate below g implying n/K will fall:

$$\begin{aligned}\frac{\dot{n}}{n} &< g\\ \frac{a(\xi K-n)}{n}N^{\beta} &< g\\ \frac{\xi K}{n}-1 &< g/aN^{-\beta}\\ \frac{\xi K}{n} &< g/aN^{-\beta}+1\\ \frac{n}{K} &> \frac{\xi}{g/aN^{-\beta}+1}\end{aligned}$$

This implies the equilibrium in the graph depicted by Figure 3 is a stable attractor in the system. Arrows indicate the direction in which the system will move.

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